Mathematics II
Finding Arc Length and the Area of a Sector
Day 1 Student Task

Arc Length
In the first task of this unit, we discussed the fact that arcs are measured in two different ways. The *measure* of an arc is calculated in units of degrees and is defined to be the measure of its central angle. Arc **length** is calculated in units of distance. In this task, you will develop a formula for calculating the length of an arc.

Consider the carousel in the picture above. The innermost horse in the picture is 12 feet from the center of the carousel. The outermost horse is 24 feet from the center.

1. Suppose the carousel makes one complete revolution.
    a. Through how many degrees does the outermost horse turn?

    b. Through how many degrees does the innermost horse turn?

    c. Do the two horses travel the same *distance*? Why or why not?

    d. If the two horses travel the same distance, how far do they travel? If they travel different distances, how far does each horse travel? Show how you know.

2. Suppose the carousel rotates through 120°.
    a. Through how many degrees does the outermost horse turn?

    b. Through how many degrees does the innermost horse turn?

    c. How far does each horse travel during this rotation? Show how you know.

3. The positions of the innermost and the outermost horses on the carousel can be modeled by two concentric circles. **Concentric** circles are coplanar circles with the same center.
    a. Use your compass to construct concentric circles that represent the positions of the innermost and outermost horses as the carousel rotates.
    b. Consider that the *distance* a horse travels is the *length* of the arc the horse traverses on its circle. Use your diagram and your answers to *Problems 1* and *2* to help you determine a formula for finding the length of any arc on any circle.

Area of a Sector

The carousel in the picture above needs refurbishing. Suppose, in an effort to make things colorful, the carnival owner wishes to paint a pattern of sectors on the carousel floor. A **sector** of a circle is a region between two radii and an arc of the circle.

4. Consider the floor of the carousel. It can be represented by the outer circle of your diagram in *Problem 3a*. Use your compass to construct a single circle that represents the floor of the carousel. What is the area of the floor? Show how you know?

5. The owner has decided to paint the floor in a repeating pattern of sectors with central angles of 10°, 20°, and then 30°. Use you protractor and a straightedge to draw the pattern on your circle. How many sectors of each degree measure are on your "floor"?

6. Suppose each sector with a central angle of 10° will be painted purple, each sector with a central angle of 20° will be painted pink, and each sector with a central angle of 30° will be painted blue. How many square feet of the floor will be painted purple? pink? blue? Show how you know.

7. Use what you have learned in *Problems 4 – 6* to help you determine a formula for finding the area of any sector of any circle.